MATHEMATICAL ABSTRACTION IN MATHEMATICS LEARNING
ENHANCING PRE-SERVICE MATHEMATICS TEACHER COMPETENCY

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Abstractions

Complex concept and with many faces

Fundamental process in mathematics education (Ferrarri, 2002)

Regarded as generalization, a process, separation process.

A process of constructing idea or general conclusion from a situation

Something related to making idea or general conclusion from a situation

Something related to creating fuzzy notion (KBBI)
A cognitive process that takes place in students’ mind, used to construct new concepts based on previous understanding.

Abstraction plays important roles in designing math instruction.
It can be used to understand student’s learning process.
It can be used to strengthen student’s math thinking skills.
Generalization, more than the process of constructing more and more inclusive classes, which require one to deal with complex symbolic expressions (Ferrari, 2002).

Conceived as a process of decontextualization, which involves moving away from perceptual or empirical aspects of a situation in which something was originally learned (Aera, 2005)

Derived from idea that abstraction as generalization implies certain degree of decontextualization (Gravemeijer, 1994)
Mathematical Abstractions

Complex concept that cannot be reduced to generalization or decontextualization only.

Both are the main components of abstraction when abstraction is viewed as part of math knowledge that already existed.

A process of vertically reorganizing some of the learner’s previous mathematical constructs within math and by math means, so as to have a construct that is new to the learner (Dreyfus, 2015)
Mathematical Abstractions

Cognitive process that occurs when someone constructs new mathematical concepts.

The construction of new knowledge happens generally during learning process.
Level of Abstractions

1. Abstraction level as the quality of the relationships between the object of thought and the thinking person.
2. Abstraction level as reflection of the process-object duality.
3. Abstraction level as the degree of complexity of the concept of thought.
The Research Questions

(1) How do pre-service math teachers’ abstraction processes take place when they learn non-conventional math concepts?

(2) What kind of math abstraction levels that could be raised by pre-service math teachers in learning non-conventional math concepts?

(3) To what extent the abstraction process of pre-service math teachers in learning non-conventional math concepts could indicate their performance in learning conventional math concepts?
The Aims of the Research

(1) To investigate the abstraction process of pre-service math teachers when they learn non-conventional math concepts

(2) To investigate math abstraction levels raised by pre-service math teachers in learning non-conventional math concepts

(3) To investigate a relationship between the abstraction process of pre-service math teachers in learning non-conventional math concepts and their performance in learning conventional math concepts
Empirical Abstraction

Types of Abstraction

Reflective Abstraction

Empirical Abstraction

Abstraction in Context (AiC)

The Interiorization

The Composition

The Encapsulation

The Generalization

- Hypothesizing
- Conflict
- Didactic Hierarchy
- Flexible transformation between models
- Argumentation
- Sophisticated inverse questions
LEVEL OF MATH ABSTRACTIONS

- Perceptual Abstraction
- Internalization
- Interiorization
- 2nd level of Interiorization
Perceptual Abstraction

- Recognizing mathematical attributes in context by utilizing physical objects
- Recognizing mathematical attributes in context based on their experiences
- Identifying previous mathematical structures (mathematical knowledge, concepts and principles) involved in context
Internalization

Representing mathematical objects into symbol or mathematical language without the presence of perceptual input

Relating previous mathematical structures with context

Manipulating abstract mathematical concepts
Interiorization

- Coordinating relationships between processes or concepts to form new understanding or new mathematical knowledge
- Operating on objects in imagination
- Projecting the concepts into other perceptual materials
2nd Level of Interiorization

- Transforming processes into objects
- Operating or manipulating the new mathematical concepts in term of symbols without material representation
- Generalizing new mathematical knowledge in different contexts
Cartesian and Parallel Coordinate Systems
(Conventional Math and Non-conventional Math Concepts)
Constructing Parallel Coordinates with 4 Dimensions
Subject: 45 Pre-service Mathematics Teachers
Duration: 6 months
Prior Knowledge Test

Subject: Attended Analytic Geometry Course
Parallel Coordinates
6 knowledge elements
Tools: AiC RBC Models

RESEARCH METHODS
Grounded Theory

To generate a theory that is grounded in data from participants who have experienced the process.

A grounded theory is not generated before the study begins.

A grounded theory is formed inductively from the data that are collected during the study itself (develop generalization after observing the data).
The data are collected primarily through one-to-one interview and observation (an ongoing process).

Data are collected and analyzed → a theory is suggested → more data are collected → the theory is revised → more data are collected → the theory is revised → more data are collected → the theory is further developed, clarified, revised. The process continues.
The theoretical and methodological Framework:

Investigating the levels of Math Abstraction process →
Battista Framework (2007) is used to differentiate levels of Abstraction of pre-service math teachers in solving Parallel Coordinate problems
Hazzan & Zaskis procedure (2005) is used to analyze abstraction level of pre-servi mathematics teachers
Grounded Theory Design

Research Procedures

- Classroom Observation
- Data Collection: Prior Test on Cartesian Coordinate
- Design for Abstraction
- Data Collection: Video Recording Task-based Clinical Interview Collection Students’ Worksheets Abstraction Test
- Data Analysis using RBC Model and Abstraction Level Theory

Case Study

- Test of Parallel Coordinates and Cartesian Coordinate Concept
- Qualitative Data Analysis using Descriptive Statistics
- Interpretation of Entire Data Analysis
Design contains activities offering the students opportunities to learn well defined mathematical ideas is required. Knowledge should not have been relevant to previous activities carried out by students (Dreyfus, et al, 2015).

Context Design for Mathematical Abstraction based on AiC Framework

Considerations in selecting and designing mathematical concepts:

- The concepts must be relatively new for abstraction for pre-service math teachers.
- The concept must not be too advanced.
- The concept must bring benefit for their mathematical horizon.
- The concept must be flexible to be embedded in curriculum.
Design of Learning Activities based on AiC Framework

- Key Concepts
- Prior Knowledge
- Specific Instructional Objectives
- Design of Activities
Micro-level in social science research referred to individual or small group of individual in particular context.

In this research the data come from dialogue of small amount of individuals.

Microanalysis is detailed line-by-line analysis necessary at the beginning of a study to generate initial categories (with their properties and dimensions) and to suggest relationships among categories (a combination of open and axial coding).

In this research, data came from participants’ social interaction in a group in the classroom context and the data analysis used a combination of open and axial coding.
Validity: the researcher checks for the accuracy of the finding by employing certain procedures.

The finding is accurate from the standpoint of the researcher, the participants, or the readers of a account (Creswell, 2009; Creswell & Miller, 2000).

Reliability: the consistency of the researcher’s approach across different researchers and different projects (Gibbs, 2007).

Generality: transferability achieved when the researcher provide sufficient information about self (the researcher as an instrument) and the research context, participants, and the researcher-participant relationship to enable the reader to decide how the finding may transfer (Fraenkel, Wallen, & Hyun, 2012).
Pre-service Mathematics Teachers’ Prior Knowledge on Cartesian Coordinate Topic

- Plotting points on Cartesian coordinate (100%)
- Examining distance between two points in Cartesian coordinate (87%)
- Defining a linear equation in Cartesian coordinate (89%)
- Defining an equation of parallel lines (74%)
- Defining an equation of two perpendicular lines (72%)
- Sketching the graph of linear function in Cartesian coordinate (74%)
This study uses RBC+C model for microanalysis of data from video transcript.

In RBC+C model, the abstraction process can be identified from the cognitive actions that appeared in the form of statements, questions, or other verbal expressions from video transcript.

The RBC+C model consists of four observable epistemic actions: Recognizing, Building-with, Construction, and Consolidation.
**Recognizing**: Includes formal and informal information that has been acquired by learner to make the situation meaningful.

**Building with**: Includes the use of prior knowledge to handle the situation at hands; refers to explicit actions of computing, sketching, justifying, reasoning, questioning, answering, and arguing.

**Construction**: Includes the process when learners become aware of the new construct more often, more confident in other situation or in process of recognizing and building with the new one.
The abstraction process of pre-service mathematics teachers in learning low-level knowledge elements take place in group contexts.

The abstraction process of pre-service mathematics teachers in learning high-level knowledge elements are in classroom contexts.

Reducing abstraction can help participants in constructing new mathematical knowledge.

The Results
The study indicates that the abstraction process in learning concept of Parallel Coordinates, as one of non-conventional concepts, has significant contribution to the performance of pre-service mathematics teachers in learning concept of Analytic Geometry.
Conclusions

Referred to the data analysis result, the abstraction level of pre-service mathematics teachers in this study can be categorized into three levels of abstraction:

level 1 (*perceptual abstraction level*)

Level 2 (*internalization level*);

Level 4 (*second level of interiorization*);

(but there are no participant belongs level 3 - *interiorization level*).
The more complex compound of the knowledge elements, the more *scaffolding* needed to help participants construct the knowledge elements in order to accomplish the epistemic actions.

Without instructional and pedagogical *scaffolding* from the lecturer, the abstraction process pertaining to the more complex knowledge elements, could not be accomplished during the learning process in the classroom.
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