On The Optimal Control Theory and Its Application to Sustainable Tourism

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Biggest Threats of Tourism Problems in Bali

1. Waste, plastic trash

2. Traffic Congestion

What is the solution of the problems ?

- Limit the number of visitor

- What is the optimal number of visitors to be sustain?

The concept of sustainable tourism

- Sustainable Tourism involves social responsibility, a strong commitment to nature and the integration of local people in any tourist operation or development. Sustainable tourism is defined by the World Tourism Organisation (WTO) and the Tourism Council (WTTC)

- How to reduce the negative impact of tourism development, so that the benefits obtained from tourism is also sustain forever

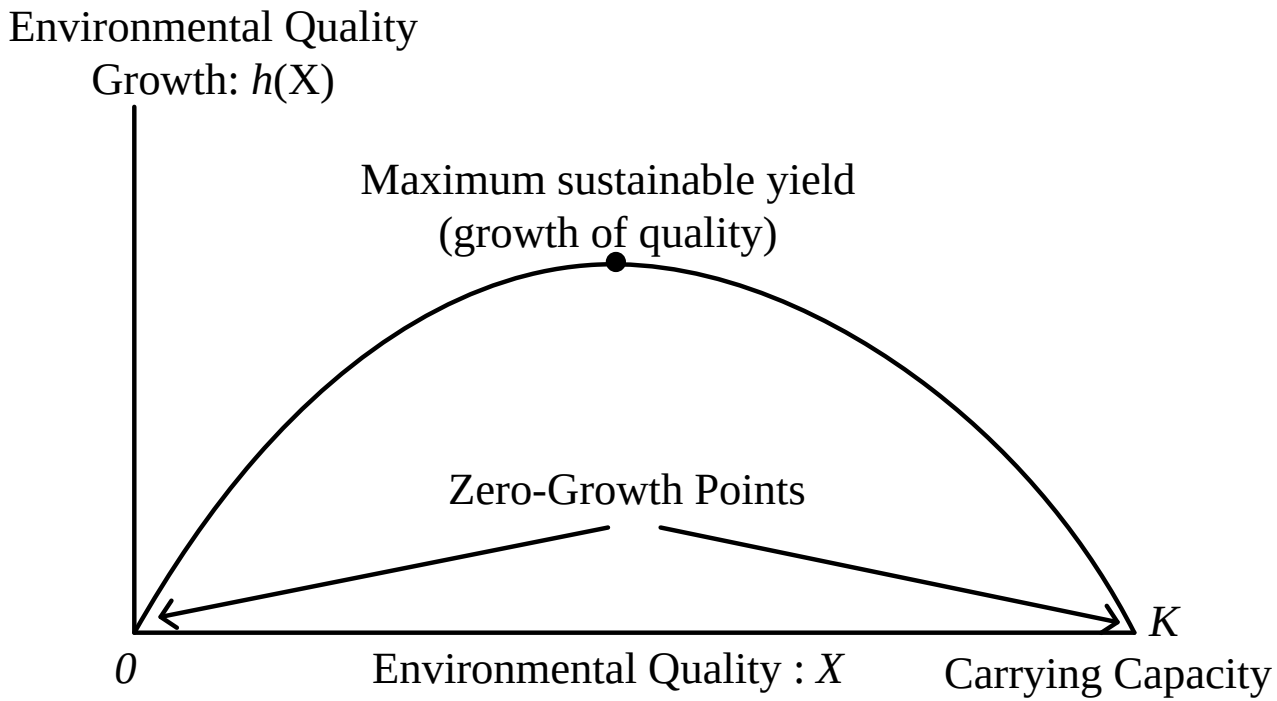
- Considering tourism as a triangular relationship between local residents, tourists and the tourism industry. (Syahid, 2016)

Mathematical model

- The growth function – using simple function $h(X)$ – implies that growth of environmental quality is a mathematical function of X

- The equation describing the total per-period change in environmental quality combines the negative influence of visitors (V) and the positive influence of natural growth $h(X)$, such that:

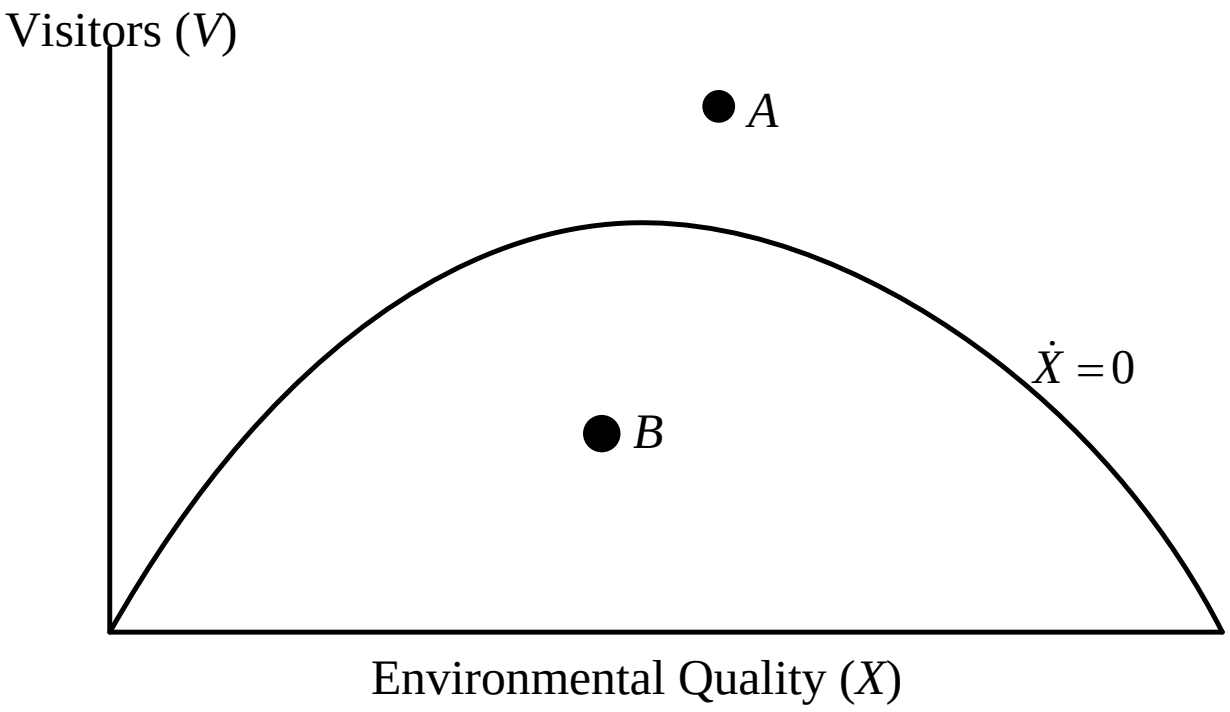
$$\dot{X} = h(X) - V$$



where $\dot{X}$ is the change in X during a time period

Environmental quality and tourism

- Sustainability of environmental quality occurs when $\dot{X} = 0$, no change in in environmental quality over time.
- point A represent conditions in which visitor damage exceeds natural renewal
- point B represent conditions in which natural renewal exceeds visitor damage – hence environmental quality will improve over time



The benefits of sustainable tourism

- Tourist industry profits may be represented as a general mathematical function of the number of visitors V and environmental quality X, given by $\Pi(V, X)$.
- The challenge facing the tourist industry is to maximizes the sum of discounted profits, $\Pi(V, X)$, over time,

$$\text{maximize} \qquad \int_0^{\infty} \Pi(V, X) e^{-rt}\, dt \qquad (1)$$

$$\text{subject to} \qquad \dot{X} = f(X) - V \qquad (2)$$

- Suppose that every visistor V reduce the quality of environment X with the rate of γ $(0 < \gamma < 1)$, this gives the relationship:

$$\dot{X} = \gamma X(t) - V \qquad (3)$$

The benefits of sustainable tourism

- Let $\alpha V(t)$ denote the economical benefit of visitors and $\beta X(t)$ denote environmental pollution created by visitors, the present velue of the net benefit given by

$$\pi(t) = \int_0^\infty \big(A(V(t)) - B(X(t)\big)e^{-rt}\, dt \tag{4}$$

- The social objective to control the the number of visitors is given by dynamic optimization of sustainable toursm:

$$\text{maximize} \quad \pi(t) = \int_0^\infty (A(V(t)) - B(X(t)))e^{-rt}\, dt \tag{5}$$

$$\text{subject to} \quad \dot{X} = \gamma X(t) - V(t) \tag{6}$$

$$X(t) \geq 0, V(t) \geq 0, X(0) \text{ given} \tag{7}$$

Solution to the dynamic optimization

- The Hamiltonian fugstion of equation (5)-(7) is given by

$$\mathcal{H} = \big(A(V(t)) - B(X(t)\big) + \lambda(t)\big(\gamma X(t) - V(t)\big) \tag{8}$$

- By the maximum principle, we have

$$\frac{\partial \mathcal{H}}{\partial V} = \frac{\partial A}{\partial V} - \lambda(t) = 0 \tag{9}$$

$$-\dot{\lambda} + \lambda = \frac{\partial \mathcal{H}}{\partial X} = \frac{\partial B}{\partial X} + \gamma\lambda(t) = 0 \tag{10}$$

- Differentiating Equation (8) with respect to t, giving

$$\dot{\lambda} = \left[\frac{\partial^2 A}{\partial V^2}\frac{\partial V}{\partial t}\right] \tag{11}$$

Solution to the dynamic optimization

- Substituting (11) into (8) giving

$$\dot{V} = \frac{\partial V}{\partial t} = \left[\frac{\partial B}{\partial X} + (-1+\gamma)\frac{\partial A}{\partial V}\right] / \left[\frac{\partial^2 A}{\partial V^2}\right] \tag{12}$$

- Differentiating the Hamiltonian function with respect to λ giving (13)

$$\dot{X} = \frac{\partial \mathcal{H}}{\partial \lambda} = \gamma X(t) - V(t) \tag{14}$$

- The case $\dot{V} = 0$ then (12) becomes

$$(\gamma - 1)\frac{\partial A}{\partial V} = -\frac{\partial B}{\partial X} \tag{15}$$

- The case $\dot{X} = 0$ then (14) becomes

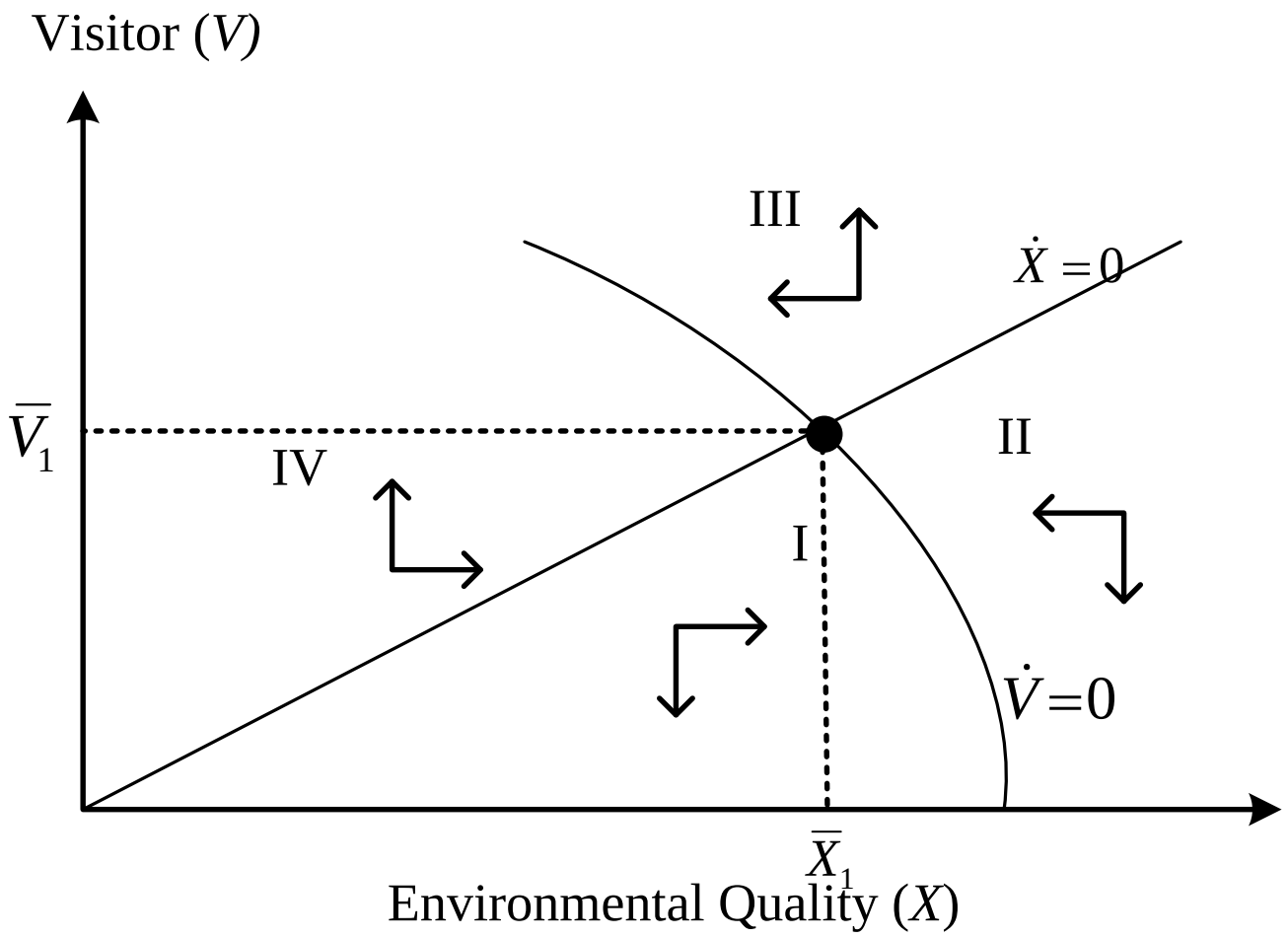
$$\gamma X(t) = V(t) \tag{16}$$

Solution to the dynamic optimization

- The total derivative of (15) and letting $\dot{V} = 0$, gives

$$\frac{\partial V}{\partial X} = \frac{\frac{\partial^2 B}{\partial X^2}}{(\gamma - 1)\frac{\partial^2 A}{\partial V^2}}$$

- The curve for the isocline $\dot{X} = 0$ is linear, as shown by (16), while the curve for the isocline $\dot{V} = 0$ has negative slove (tangent). The intersection of these two curve is known as steady-state solution.



Interpretation of the solution

- To the left of curve $\dot{X} = 0$, maximization of profits requires increasing visitors over time. To the right of this curve, maximization requires decreasing visitors over time.
- The intersection of $\dot{V} = 0$ and $\dot{X} = 0$, point $(\bar{X}_1, \bar{V}_1)$, represents the point at which the present value of tourism industry profits are at their maximum possible level over time, given the constraint that both the number of visitors and environmental quality are sustained at a fixed level indefinitely.
- The dynamic model presented here allows one to identify feasible, sustainable solutions that maximize benefits to tourism, or even to the local residents.

Conclusion

- This discussion presents an optimal control model in a dynamic model that is common in the case of tourism which refers to the same model as models that are often used as references in modeling of renewable natural resources.

- The model is meant to provide a preliminary step towards greater structure and clarity in the discussion of tourism sustainability.

- This model is only an alternative model in analyzing sustainable tourism. It is also a conceptualization of environmental damage that occurs implicitly in order to realize sustainable tourism and provides a broader understanding of what is implied and what is not implied in building eco-friendly tourism.

Thank You